This exercise set deals with the skyline problem: Given a set of rectangles in the Cartesian plane such that the base of each rectangle coincides with the x-axis, determine the upper boundary of the union of the rectangles. (If the rectangles were cross-sectional views of buildings, as seen arranged along a stretch of flat terrain from some remote fixed perspective, the upper boundary would be the polyline separating the ground and the buildings from the sky above them.)

You may assume that each rectangle is given as a triple of real numbers \(x_1, y_1, x_2\), where \(x_1\) is the abscissa of the left-hand corners of the rectangle, \(y_1\) the ordinate of its upper corners, and \(x_2\) the abscissa of its right-hand corners. (So in valid input \(x_1 < x_2\) and \(y_1 > 0\).) The program should compute and return the skyline as an odd-length list of real numbers \(x_1, y_1, x_2, y_2, \ldots, x_{n-1}, y_{n-1}, x_n\), where the boundary begins at \((x_1, 0)\), goes up to \((x_1, y_1)\), then rightwards to \((x_2, y_1)\), up or down to \((x_2, y_2)\), rightwards to \((x_3, y_2)\), and so on, ending with line segments from \((x_{n-1}, y_{n-1})\) to \((x_n, y_{n-1})\) and from there to \((x_n, 0)\).

So, for instance, given the rectangles

\[
\begin{align*}
(9.1, 7.7, 11.2) \\
(26.6, 5.6, 31.5) \\
(7.7, 5.6, 13.3) \\
(15.4, 10.5, 16.8) \\
(30.1, 7.0, 33.6) \\
(4.9, 4.2, 23.8) \\
(28.7, 2.1, 30.8) \\
(17.5, 2.8, 25.2) \\
(16.1, 8.4, 18.9) \\
(1.4, 8.4, 6.3) \\
(18.2, 5.6, 20.3)
\end{align*}
\]

the program would construct and return the list \((1.4, 8.4, 6.3, 4.2, 7.7, 5.6, 9.1, 7.7, 11.2, 5.6, 13.3, 4.2, 15.4, 10.5, 16.8, 8.4, 18.9, 5.6, 20.3, 4.2, 23.8, 2.8, 25.2, 0.0, 26.6, 5.6, 30.1, 7.0, 33.6)\).

The problem is non-trivial because the rectangles can overlap in many different ways, and even be obscured entirely. (For instance, the seventh rectangle in the list above contributes nothing to the skyline, because every part of it is overlapped by some taller rectangle.)

**Exercises**

0. In Scheme, C, or Java, design, implement, and test a program that solves any specified instance of the skyline problem.

1. Classify your program according to the asymptotic behavior of its running time as a function of the number of rectangles in the input.

These exercises will be due at the beginning of class on Wednesday, December 7. Please submit a legible hard copy of your solutions at that time.

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