Exercise set #1
CSC/MAT 208, “Discrete Structures”
Department of Computer Science
Grinnell College
September 10, 2018

These exercises will be due at the beginning of class on Monday, September 17.

To submit the exercise, log in on MathLAN and open a terminal window, create a new directory within your home directory, copy or move the files you want to submit into that directory, and then run the command

```
/home/reseda/executables/submit-208 directory-name
```
putting the name of the directory you created in place of directory-name.

1. Prove that, for any positive integers \( a \) and \( b \), if \( a \) evenly divides \( b \) and \( b \) evenly divides \( a \), then \( a = b \).

2. One of the simplest equations involving the summation operator is

\[
\sum_{i=0}^{n} i = \frac{n^2 + n}{2},
\]
which says that the sum of the natural numbers from 0 up to and including \( n \) is \((n^2 + n)/2\). This equation is true for every natural number \( n \). There is an easy proof of this fact by mathematical induction, and it is Theorem 2.2.1 in our textbook, where the authors offer a different proof, by contradiction, using an axiom called the “well-ordering principle” (pages 31 and 32).

   (a) Using this theorem, prove that

\[
\sum_{i=0}^{2n} i = 2n^2 + n
\]

(that is, “The sum of the natural numbers from 0 up to and including \( 2n \) is \( 2n^2 + n \”).

   (b) Using the same theorem, prove that

\[
\sum_{i=0}^{n} 2i = n^2 + n
\]

(“The sum of the doubles of the natural numbers from 0 up to and including \( n \) is \( n^2 + n \”).

   (c) Find a formula that expresses the sum of the odd natural numbers from 1 up to and including \( 2n + 1 \), write a summation equation to state this generalization, Using the results of parts (a) and (b), prove that your equation is true for every natural number \( n \).

3. Using the summation procedure that you developed in the lab “Libraries in R7RS Scheme,” write a Scheme procedure `double-summation-of-products` that takes any natural number \( n \) as an argument and computes and returns

\[
\sum_{i=0}^{n} \sum_{j=0}^{n} ij.
\]
Use your procedure to compute the numerical value of this double summation when \( n \) is 0, 1, 2, 3, 4, and 5.

For extra credit, express the value of the summation expression as a polynomial function of \( n \) and prove that your formula is correct for every natural number \( n \).