Exercise set #2
CSC/MAT 208, “Discrete Structures”
Department of Computer Science
Grinnell College
September 28, 2018

These exercises will be due at the beginning of class on **Friday, October 5**.
To submit the exercise, log in on MathLAN and open a terminal window, create a new directory within your home directory, copy or move the files you want to submit into that directory, and then run the command

```
/home/reseda/executables/submit-208 directory-name
```
putting the name of the directory you created in place of `directory-name`.

1. Using the axioms and rules of inference of the propositional calculus, prove that

   \[(P \rightarrow Q) \leftrightarrow ((P \vee Q) \leftrightarrow Q)\]

   is a theorem.

2. Using the axioms and rules of inference of the predicate calculus, prove that

   \[(\forall x \exists y (P(x) \land Q(y)) \leftrightarrow (\forall x P(x) \land \exists y Q(y)))\]

   is a theorem.

3. Using the axioms and rules of inference of the theory of elementary arithmetic (which includes the theory of identity and the predicate calculus), prove that multiplication of natural numbers is commutative. (Hint: You may want to identify and prove some related propositions as lemmas first.)

4. Using the axioms and rules of inference of the propositional calculus and the additional axioms listed in the “Proofs about Sets” lab, prove that

   \[\forall x \forall y (x \cup (y - x)) = (x \cup y)\]

   is a theorem.