Exercise set #3 (revised)
CSC/MAT 208, “Discrete Structures”
Department of Computer Science
Grinnell College
November 14, 2018

Exercises 1–3 will be due at the beginning of class on Monday, November 19. Exercise 5 will be due at the beginning of class on Wednesday, November 21.

To submit the exercise, log in on MathLAN and open a terminal window, create a new directory within your home directory, copy or move the files you want to submit into that directory, and then run the command

/home/reseda/executables/submit-208 directory-name

putting the name of the directory you created in place of directory-name.

1. In each hand of the card game Skat, ten cards are dealt to each player from a deck containing thirty-two cards (ace, king, queen, jack, ten, nine, eight, and seven in each of four suits — spades, hearts, clubs, and diamonds). How many different possible Skat hands are there?

2. When a Skat hand contains no cards of a particular suit, it is said to be void in that suit. How many Skat hands are void in spades or hearts (or both)?

3. Prove, by a combinatorial argument, that

\[ S(n+1, m+1) = \sum_{k=0}^{n} \left( \binom{n}{k} \cdot S(k, m) \right) \]

for all natural numbers \( m \) and \( n \). (Here ‘\( S \)’ indicates a Stirling number — recall that \( S(i, j) \) is the number of \( j \)-compartment partitions of a set of cardinality \( i \).)

4. (For extra credit only.) Define and test a Scheme procedure that takes any natural number \( n \) as argument and returns a set containing all of the integer partitions of \( n \).

5. A breakout for a natural number \( n \) is a bag of natural numbers that add up to \( n \). For example, the bag \([5, 1, 1, 0]\) is a breakout of 7, because \( 5 + 1 + 1 + 0 = 7 \).

   Obviously, there are infinitely many breakouts of any natural number, because any breakout can be extended by adding another 0 to the bag. But there is still possible to count breakouts of a specific natural number \( n \) of a fixed bag cardinality \( k \). For example, there are exactly eleven four-member breakouts of 7, namely \([7, 0, 0, 0]\), \([6, 1, 0, 0]\), \([5, 2, 0, 0]\), \([5, 1, 1, 0]\), \([4, 3, 0, 0]\), \([4, 2, 1, 0]\), \([4, 1, 1, 1]\), \([3, 3, 1, 0]\), \([3, 2, 2, 0]\), \([3, 2, 1, 1]\), and \([2, 2, 2, 1]\).

   (a) For any natural numbers \( n \) and \( k \), let \( K(n, k) \) be the number of \( k \)-member breakouts of \( n \). Write a recursive definition of the function \( K \) and provide a combinatorial justification for your definition.

   (b) Design, write, and test a Scheme procedure \texttt{breakouts} that takes two natural numbers \( n \) and \( k \) as arguments and returns a set containing all of the \( k \)-member breakouts of \( n \).

   (c) The “shape” of a Skat hand is a bag that contains the number of cards in each of the four suits in that hand. For example, the shape of a Skat hand that comprises five spades, one heart, three diamonds, and one club is \([5, 1, 3, 1]\). Determine how many different shapes a Skat hand can have.

Copyright © 2018 John David Stone

This work is licensed under the Creative Commons Attribution-ShareAlike 4.0 International License. To view a copy of this license, visit

http://creativecommons.org/licenses/by-sa/4.0/deed.en

or send a letter to Creative Commons, 543 Howard Street, 5th Floor, San Francisco, California, 94105, USA.