Exercise set #4
CSC/MAT 208, “Discrete Structures”
Department of Computer Science
Grinnell College
April 19, 2019

Your solutions to these exercises will be due at the beginning of class on Friday, April 26.
To submit them, log in on MathLAN and open a terminal window, create a new directory
within your home directory, copy or move the files you want to submit into that directory, and then
run the command

/home/reseda/executables/submit-208 directory-name

putting the name of the directory you created in place of directory-name.

1. Prove by mathematical induction that, for every positive integer n, the number of set
partitions of an n-element set into exactly two compartments is $2^{n-1} - 1$.

2. Determine, as a function of n, how many relations on an n-element domain are asymmetric.
(Hint: For any distinct elements a and b of the domain, an asymmetric relation may contain ⟨a, b⟩,
⟨b, a⟩, or neither, but may not contain both of those ordered pairs.)

3. A distribution of a natural number n into k parts is a bag containing k natural numbers
whose sum is n. For example, [5, 2, 1, 1, 0] is a distribution of 9 into five parts. (Note that
distributions differ from integer partitions in that a distribution can have 0 as an element and an
integer partition cannot.)

Use a combinatorial argument based on the Bijection Rule to prove that the number of dis-
tributions of n into k parts is I(n + k, k) (where I is the function defined in the handout “Bell
Numbers and Integer Partitions” to count integer partitions).

4. A one-way-unbounded tape scanner is a device with a reading head that can pick up
information from a paper tape that is fed into it. The tape is divided into byte-sized cells, and the
reading head must be positioned on the first cell of the tape before a scan is started.

Each cycle of the operation of the scanner consists of reading and reporting out the contents
of the cell that the reading head is currently on, and then advancing or retracting the tape so that
the reading head is positioned on one of the adjacent cells, with the restriction that the scan fails
if the tape is retracted while the reading head is on the first cell of the tape. (So, for example,
when the scanner is first started, after it reads the contents of the first cell of the tape, it must
then advance to the second cell; if it tried to retract the tape, the scan would fail.)

A movement pattern for the scanner is a sequence of advances and retractions of the tape
that does not result in a scan failure and ends with the reading head positioned once again on the
first cell of the tape. (Thus a movement pattern must contain the same number of advances and
retractions and so must be of even length.)

Show that, for any natural number n, the number of different movement patterns of length 2n
is the Catalan number $C_n$.

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